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Insurance-markets Equilibrium with Sequential Non-convex Market-Sector and Divisible Informal-Sector Labor Supply

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Abstract. *This paper describes the lottery- and insurance-market equilibrium in an economy with non-convex market-sector employment and informal sector work. In contrast to Vasilev 2016a, the discrete-continuous labor supply decision in this paper is a sequential one, and instead of home production, we focus on informal activity. The presence of non-convexity requires that an insurance market for market-sector employment be put in operation to achieve market completeness. In addition, given that the labor choice for market work and informal-sector hours is made in succession, the insurance market for market employment needs to close before the labor supply choice in the grey economy is made. This timing is reminiscent of the results obtained in Vasilev 2016b) and also a direct consequence of the sequential nature of the discrete-continuous sectoral labor supply decision.*

Keywords: Indivisible labour, Lotteries, Discrete-continuous mix, Informal economy, Insurance.

JEL Classification Codes: E1, J22, J46.

1. Introduction and Motivation

The purpose of this paper is to explore the problem of non-convex labor supply decision in an economy with both discrete and continuous labor decisions. The novelty relative to those studies is that the earlier setups (Vasilev 2016a,b) were dealing with private-public sector and full-time work vs. overtime, respectively, while here the focus is on the market vs. unofficial work margin. The partucular focus in this paper is on the lottery- and insurance-market equilibrium in an economy with non-convex market-sector employment and informal sector work. In contrast to Vasilev

Vasilev 2016a, the discrete-continuous labor supply decision in this paper is a sequential one, and instead of home production, we focus on informal activity. The presence of non-convexity requires that an insurance market for market-sector employment be put in operation to achieve market completeness. In addition, given that the labor choice for market work and informal-sector hours is made in succession, the insurance market for market employment needs to close before the labor supply choice in the grey economy is made. This timing is reminiscent of the results obtained in Vasilev Vasilev 2016b and also a direct consequence of the sequential nature of the discrete-continuous sectoral labor supply decision.

2. Model Setup

The basis of the model is the setup described in Vasilev 2017, but extended to incorporate an institution selling unemployment insurance. The economy is static, there is no physical capital, and agents face a sequential discrete-continuous labor supply decision. There is a large number of identical one-member households, indexed by i and distributed uniformly on the $[0, 1]$ interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index i to save on notation.

2.1 Description of the model

Each one-member household maximizes the following utility function:

$$U(c, l) = \ln c + \alpha \ln l, \tag{1}$$

where

- c denotes consumption of market output,
- l is the leisure enjoyed by each individual household,
- $\alpha > 0$ is the relative weight attached to utility of leisure.

Each household is endowed with a time endowment of unity, which can be split between hous worked in either the official sector, h_m, hours worked in the informal economy ("black market"), h_b, and leisure l, so that

$$h_m + h_b + l = 1 \tag{2}$$

The households make a sequential labor supply choice: The first is whether to work full-time in the market sector, or not at all. In other words,

$$h_m \in \{0; \bar{h}\}.$$

Conditional on not working in the market sector, a household may decide to go and work in the grey sector, where it can supply any number of hours, i.e.

$$h_b \in [0, 1].$$

That is, the first labor choice is indivisible, while the second is divisible. Also, it will be assumed that

$$h_b = 0$$

whenever

$$h_m = \bar{h},$$

or a household employed full-time in the market sector would choose not to supply any hours in the grey economy. This assumption is put in place to guarantee that each worker can only participate in one of the production sectors. Next, the hourly wage rate in the official ("market") sector and the implicit rate in the informal economy ("black market") sectors are denoted by w^m and w^b, respectively. Finally, the households own the firm in the market economy, and are entitled an equal share of the profit (π).

The problem faced by a household that decides to work full-time in the market sector is then to set

$$h_m = \bar{h}$$

and enjoy

$$U^m = \ln(w^m \bar{h} + \pi) + \alpha \ln(1 - \bar{h}), \tag{3}$$

while a household that decides not to work in the market sector chooses

$$h_b \in [0, 1]$$

to maximize its utility function

$$\max_{h_b} U^b = \ln(w^b h_b + \pi) + \alpha \ln(1 - h_b). \tag{4}$$

The optimal labor choice in the grey economy is then characterized by the following first-order condition:

$$\frac{w^b}{w^b h_b + \pi} = \frac{\alpha}{1 - h_b}, \tag{5}$$

or

$$h_b = \frac{w^b - \alpha\pi}{(1 + \alpha) w^b} \tag{6}$$

That is, optimal choice of hours worked in the informal economy is a function of both the wage and profit rate in the official sector, which the household takes as given.

2.2 Stand-in firm: market sector

There is a representative firm in the model economy, which operates in the market sector. It produces a homogeneous final product using a production function that requires labor H_m as the only input. For simplicity, output price will be normalized to unity. The production function $f(H_m)$ features decreasing returns to scale and satisfies the following conditions:

$$f'(H_m) > 0,$$

$$f''(H_m) < 0,$$

$$f'(0) = \infty,$$

$$f'(\bar{h}) = 0.$$

The representative firm acts competitively by taking the wage rate w^m as given, and chooses H_m to maximize profit:

$$\pi = f(H_m) - w^m H_m \quad \text{s.t.} \quad 0 \leq H_m \leq \bar{h}. \tag{7}$$

In equilibrium, there will be positive profit, which follows from the assumptions imposed on the production function.

2.3 Stand-in firm: unofficial sector

Each worker in the unofficial sector has access to an individual concave production function ("backyard technology") that uses only labor, $g(h_b)$, where

$$g'(h_b) > 0,$$

$$g''(h_b) < 0,$$

$$g'(0) = \infty,$$

$$g'(1) = 0.$$

Each firm in the unofficial sector will then hire labor h_b to maximize static profit

$$\max_{h_b} g(h_b) - w^b h_b \quad \text{s.t.} \quad 0 \leq h_b \leq 1. \tag{8}$$

With free entry, profits in the sector equal to zero, hence the implicit wage w^b in the unofficial sector equals the average product of labor, i.e.

$$w^b = \frac{g(h^b)}{h_b}. \tag{9}$$

3. Insurance Market: Market-sector Insurance Company

An alternative way to represent the labor selection arrangement in the market sector is to regard workers as participants in a lottery with the proportion employed equal to the probability of being selected for work. Therefore, we can introduce insurance markets, and allow households to buy insurance, which would allow them to equalize the actual income received independent of the employment status in the market sector. More specifically, the structure of the insurance industry is as follows: there is one representative insurance company for market sector employment, which services all households and maximizes profit. It receives revenue if a household is working in the market sector and makes payment if it is not. At the beginning of the period, the households decide if and how much insurance to buy against the probability of being chosen for market-sector work. Insurance costs q^m per unit, and provides one unit of income if the household is not employed in the market sector. Thus, household will also choose the quantity of insurance to purchase b^m; we can think of insurance as bonds that pay out only in case the household is not chosen for work in the official economy. Then, the company closes, and the households not selected for official-sector work, choose how many hours to supply in the informal sector.

The amount of insurance sold by the insurance company is a solution to the following problem: Taking $q^m(i)$ as given, $b^m(i)$ solves

$$\max_{b^m(i)} \lambda^m(i)q^m(i)b^m(i) - [1 - \lambda^m(i)]b^m(i) \tag{10}$$

with free entry profits are zero, hence

$$\lambda^m(i)q^m(i)b^m(i) - [1 - \lambda^m(i)]b^m(i) = 0, \tag{11}$$

hence the insurance market for each household clears.

4. Decentalized Competitive Equilibrium (DCE) with lotteries

4.1 Definition of the DCE with lotteries

A competitive Equilibrium **with Lotteries** for this economy is a list

$$(c^m(i), c^b(i), \lambda^m(i), h_b(i), p, w^m, w^b, \pi) \tag{12}$$

such that the following conditions are fulfilled.

1. **Consumers maximization condition.** Taking prices w^m, w^b, π as given, for each i, the sequence

$$\sigma = (c^m(i), c^b(i), \lambda^m(i), h_b(i)) \tag{13}$$

solves the maximization problem

$$\max_{\sigma\in\Sigma} \quad \lambda^m(i)\ln c^m(i) + (1-\lambda^m(i))\ln c^b(i) + \lambda^m(i)\alpha\ln(1-\bar{h}) + \\ +(1-\lambda^m(i))\alpha\ln(1-h_b(i)) \tag{14}$$

s.t

$$\lambda^m(i)c^m(i) + (1-\lambda^m(i))c^b(i) = \\ = \lambda^m(i)w^m\bar{h} + (1-\lambda^m(i))w^b h_b(i) + \pi, \tag{15}$$

with

$$c_m(i) \geq 0, \quad c^b(i) \geq 0, \quad 0 < \lambda^m(i) < 1, \quad \forall i, \tag{16}$$

where Σ is the constraint defined by relations (15)-(16).

2. **Market-sector firm maximization condition**. Taking prices w^m, w^b, π as given,

$$\max_{\bar{H}^m\geq 0} F(\bar{H}^m) - w^m\bar{H}^m. \tag{17}$$

3. **Informal-sector firm maximization condition**. Taking prices w^m, w^b, π as given,

$$\max_{h^b(i)\geq 0} g(h^b(i)) - w^b h^b(i) \quad s.t \quad g(h^b(i)) - w^b h^b(i) = 0, \quad \forall i. \tag{18}$$

4. **Market-clearing condition**: We have

$$\int_i \lambda^m(i)\bar{h}di = \bar{H}^m, \tag{19}$$

$$\int_i \{\lambda^m(i)c^m(i) + (1-\lambda^m(i))c^b(i)\}di = F(\bar{H}^m) + \int_i (1-\lambda^m(i))g(h_b(i))di, \tag{20}$$

where the last equation describes clearing in the goods market. Note that in line with national income accounting, output from informal activity also counts towards total gross domestic product.

4.2 Characterizing the DCE

The household's problem is as follows:

$$\mathcal{L} = \max_{\sigma\in\Sigma} \lambda^m(i)\ln c^m(i) + (1-\lambda^m(i))\ln c^b(i) + \lambda^m(i)\alpha\ln(1-\bar{h}) + \\ (1-\lambda^m(i))\alpha\ln(1-h_b(i)) - \mu[\lambda^m(i)c^m(i) + (1-\lambda^m(i))c^b(i) + \\ -\lambda^m(i)w^m\bar{h} - (1-\lambda^m(i))w^b h_b(i) - \pi], \tag{21}$$

where μ is the Lagrangian multiplier in front of the households' budget constraint. The first-order optimality conditions are as follows:

$$c^m(i) : \frac{1}{c^m(i)} = \mu, \quad \forall i, \tag{22}$$

$$c^b(i) : \frac{1}{c^b(i)} = \mu, \quad \forall i. \tag{23}$$

It follows that

$$c = c^m(i) = c^b(i) = 1/\mu, \quad \forall i. \tag{24}$$

We simplify the Lagrangian by suppressing all consumption superscripts and i notation in the derivations to follow

$$h_b(i) : \frac{\alpha}{1 - h_b(i)} = \mu w^b, \quad \forall i, \tag{25}$$

$$\lambda^m(i) : \alpha[\ln(1 - \bar{h}) - \ln(1 - h_b(i))] = \mu[w^b h_b(i) - w^m \bar{h}], \quad \forall i. \tag{26}$$

The former condition states that the marginal rate of substitution between labor in the informal sector and consumption equals the implicit sector in the black market. The latter says that the difference from enjoying leisure (given that consumption utility has been equilizes across states equals the difference in the labor income, multiplied by the shadow price of consumption (i.e. expressed in consumption utility terms). This implicitly characterizes optimal market sector participation rate λ^m. Note that it is optimal from the benevolent planner/government point of view to choose randomly λ^m and to introduce uncertainty. With randomization, choice sets are convexified, and thus market completeness is achieved. Now we extend the commodity space to include insurance markets explicitly.

5. Decentralized Competitive Equilibrium (DCE) with insurance markets

5.1 Definition of the DCE with insurance markets

A competitive Equilibrium **with Lotteries and insurance markets** for this economy is a list

$$(c^m(i), c^b(i), \lambda^m(i), b^m(i), q^m(i), h_b(i), p, w^m, w^b, \pi), \tag{27}$$

such that the following conditions are fulfilled.

1. **Consumers maximization condition.** Taking prices p, w^m, w^b, π as given, for each i, the sequence

$$\sigma = (c^m(i), c^b(i), \lambda^m(i), h_b(i)) \tag{28}$$

solves the maximization problem

$$\max_{\sigma\in\Sigma} \lambda^m(i)[\ln c^m(i) + \alpha \ln(1-\bar{h})] + (1-\lambda^m(i))[\ln c^b(i) + \alpha \ln(1-h_b(i))] \quad (29)$$

s.t

$$pc^m(i) + b^m(i)q^m(i) = w^m\bar{h} + \pi \quad (30)$$
$$pc^b(i) = b^m(i) + w^b h_b(i) + \pi \quad (31)$$
$$c^m(i) \geq 0, c^b(i) \geq 0, 0 < \lambda^m(i) < 1, \forall i \quad (32)$$

or

$$pc^m(i) + pq^m(i)c^b(i) = w^m\bar{h} + q^m(i)w^b h^b(i) + (1+\pi)q^m(i) \quad (33)$$

where Σ is the constraint defined by the above relations.

2. **Market-sector firm maximization condition**. Taking prices w^m, w^b, π as given,

$$\max_{\bar{H}^m \geq 0} F(\bar{H}^m) - w^m\bar{H}^m. \quad (34)$$

3. **Informal-sector firm maximization condition**. Taking prices w^m, w^b, π as given, $\forall i$

$$\max_{h^{b(i)}\geq 0} g(h^b(i)) - w^b h^b(i) \quad s.t \quad g(h^b(i)) - w^b h^b(i) = 0. \quad (35)$$

4. **Insurance-company** Taking $g^m(i)$ as given, $b^m(i)$ solves

$$\max_{b^m(i)} \lambda^m(i)q^m(i)b^m(i) - (1-\lambda^m(i))b^m(i) \quad (36)$$

with free entry profits are zero, hence

$$\lambda^m(i)q^m(i)b^m(i) - (1-\lambda^m(i))b^m(i) = 0. \quad (37)$$

This implicitly clears the insurance market for each individual in the market sector.

5. **Market-clearing condition**: We have

$$\int_i \lambda^m(i)\bar{h}di = \bar{H}^m, \quad (38)$$
$$\int_i \{\lambda^m(i)c^m(i) + (1-\lambda^m(i))c^b(i)\}di = F(\bar{H}^m) + \int_i (1-\lambda^m(i))g(h_b(i))di. \quad (39)$$

5.2 Characterization of the DCE with insurance markets

$$\mathcal{L} = \max_{\sigma\in\Sigma} \lambda^m(i)\{\ln c^m(i) + \alpha \ln(1-\bar{h}) + (1-\lambda^m(i))[\ln c^b(i) + \alpha\ln(1-h_b(i))\} + \\ -\mu[pc^m(i) + pq^m(i)c^b(i) - w^m\bar{h} - q^m(i)w^b h^b(i) - (1+\pi)q^m(i)] \quad (40)$$

Normalize $p = 1$.

$$c^m(i) : \frac{\lambda^m(i)}{c^m(i)} = p\mu, \quad \forall i, \quad (41)$$

$$c^b(i) : \frac{(1-\lambda^m(i))}{c^b(i)} = pq^m(i)\mu, \quad \forall i. \quad (42)$$

Optimal λ^m $(\lambda^m(i) = \lambda^m, \forall i)$ is implicitly characterized by the zero-profit condition from the market-sector insurance company:

$$\frac{\lambda^m}{1-\lambda^m} = \frac{1}{q^m}, \quad (43)$$

which implies that the price of the insurance equals the ratio of probabilities of the two events ("the odds ratio"). Combining this with the other optimality condition, we obtain that households buy full insurance to equalize consumption,

$$c^m = c^b, \quad \forall i.$$

That is, in the presence of uncertainty, we need insurance companies to achieve market completeness.

6. Conclusions

This paper describes the lottery- and insurance-market equilibrium in an economy with non-convex market-sector employment and informal sector work. The presence of non-convexity requires that an insurance market for market-sector employment be put in operation to achieve market completeness. In addition, the insurance market for market employment needs to close before the labor supply choice in the grey economy is made. This timing of the insurance-market operation is a direct consequence of the sequential nature of the discrete-continuous labor supply decision.

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